SEPARABLE DIFF EQ SOLUTIONS

1a) $\frac{dy}{dx} = \frac{x^2}{y^2}$

$\int y^2\,dy = \int x^2\,dx$

$\frac{y^3}{3} = \frac{x^3}{3} + C_1$

$y^3 = x^3 + C_2 \quad , \quad C_2 = 3C_1$

$y = (3x^3 + C_2)^{1/3}$

1b) $y' = x^2 y$

$\int \frac{dy}{y} = \int x^2\,dx$

$\ln|y| = \frac{x^3}{3} + C_1$

$e^{\ln|y|} = e^{\left(\frac{x^3}{3} + C_1\right)}$

$|y| = e^{x^3/3} + C_1$

$y = \pm e^{x^3/3} \cdot C_2 \quad , \quad C_2 = e^{C_1}$

2a) $x + 3y^2\sqrt{x^2+1}\,\frac{dy}{dx} = 0 \qquad y(0) = 1$

$$\int 3y^2 dy = \int \frac{-x\,dx}{\sqrt{x^2+1}} \quad \Rightarrow \quad u = x^2+1, \; du = 2x\,dx$$

$$\frac{3y^3}{3} = -\frac{1}{2}\int u^{1/2}\,du$$

$$y^3 = -\frac{1}{2}\,\frac{u^{1/2}}{1/2} + C$$

$$y^3 = -\sqrt{x^2+1} + C$$

$$y = \left(-(x^2+1)^{1/2} + C\right)^{1/3}$$

I.C. $\quad 1 = \left(-(1)^{1/2} + C\right)^{1/3}$

$$1 = -1 + C$$

$$C = 2$$

$$\therefore \quad y = \left(-(x^2+1)^{1/2} + 2\right)^{1/3}$$

2b) $\frac{dp}{dt} = \sqrt{pt}$, $p(1) = 2$

$$\int \frac{dp}{\sqrt{p}} = \int \sqrt{t}\,dt$$

$$\frac{p^{1/2}}{1/2} = \frac{t^{3/2}}{3/2} + C_1$$

$$p^{1/2} = \frac{1}{3}t^{3/2} + C_2, \quad C_2 = \frac{1}{2}C_1$$

$$p = \left(\frac{1}{3}t^{3/2} + C_2\right)^2$$

I.C $\quad p(1) = 2 = \left(\frac{1}{3} + C_2\right)^2$

$$\sqrt{2} = \frac{1}{3} + C_2 \quad \Rightarrow \quad C_2 = -\frac{1}{3} + \sqrt{2}$$

$$\therefore \; p = \left(\frac{1}{3}t^{3/2} - \frac{1}{3} + \sqrt{2}\right)^2$$

3) $\frac{dy}{dt}$ = rate in − rate out

rate in = $0.03 \frac{kg}{L} \cdot 25 \frac{L}{min} = 0.75 \ kg/min$

rate out = $\frac{y(t) \ kg}{6000 L} \cdot \frac{25 L}{min}$

$= \frac{y(t)}{200}$

$\frac{dy}{dt} = \frac{3}{4} - \frac{y}{200}$

$\frac{dy}{dt} = \frac{150 - y}{200}$

$\frac{dy}{150-y} = \frac{dt}{200} \Rightarrow$ $u = 150-y$, $du = -dy$

$-\ln|150-y| = \frac{1}{200} t + C$

take $e^{\wedge}$ both sides $\Rightarrow |150-y| = e^{-t/200} \cdot e^{-C}$

$|150-y| = e^{-t/200} \cdot k$, $k = e^{-C}$

$\pm(150-y) = ke^{-t/200}$

(+ve) case $\Longleftarrow$ Cant have (−ve) salt, so use (+ve) case

$150-y = ke^{-t/200}$

$y(0)=20 \Rightarrow 150-20 = k$

$k = 130$

$150-y = 130e^{-t/200}$

$y = -130e^{-t/200} + 150$

$y(30) = -130e^{-30/200} + 150$

$= 38.1 \ kg$

4)

$$xy = k$$

$$y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

negative inverse slope $\Rightarrow$ $\frac{dy}{dx} = \frac{x}{y}$

$$\int dy \cdot y = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\frac{y^2}{2} - \frac{x^2}{2} = C$$

$$y^2 - x^2 = k, \qquad k = 2C$$