Math 1B: Calculus

(Spring 2020)

Discussion 23: Separable Equations

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- 1st order differential equations which can be solved explicitly
- of the form $\frac{dy}{dx} = g(x)f(y)$
- gather x-terms and y-terms and integrate -> $\int \frac{1}{f(y)} dy = \int g(x) dx$, where $f(y) \neq 0$
- 1. Solve the separable differential equation

a.
$$\frac{dy}{dx} = \frac{x^2}{y^2}$$

b.
$$y' = x^2 y$$

2. Solve the separable initial value differential equation

a.
$$x + 3y^2 \sqrt{x^2 + 1} \frac{dy}{dx} = 0$$
, I.C. $y(0) = 1$

b.
$$\frac{dp}{dt} = \sqrt{pt}$$
, I.C. $p(1) = 2$

3. A tank contains 20kg of salt dissolved in 5000L of water. At the tank inlet, 0.03kg of salt/L enters the tank at a flow rate of 25L/min. The brine is thoroughly mixed and flows out of the tank at the same flow rate.

After ½ hour, how much salt is in the tank?



4. Show that the orthogonal trajectories of the family of hyperbolas xy = k are the hyperbolas given by $x^2 - y^2 = k$