Quiz 5

Instructions

This quiz is **optional**. It will be graded purely so you may get some more detailed feedback. Use whatever resources you need, but, at least for your first attempt, you might consider trying to model an exam environment.

Your solutions are due by **11:59 PM on Wednesday, March 18** via Gradescope; there is no guarantee that late submissions will be graded. You should use a fresh sheet of paper for your solutions. Since you have time to **revise** your solutions, make sure your submitted work is **clear** and **concise**. You should try to write solutions that a fellow student would be able to follow.

1. (3 points) Does the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{26 - 10n + n^2}$ converge or diverge?

2. (3 points) Suppose $\{b_n\}$ is a sequence of positive numbers such that $\lim\limits_{n\to\infty} b_n = 2$. For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{b_1 b_2 \cdots b_n}$ converge?

3. (3 points) Does the series $\sum_{n=2}^{\infty} \frac{1}{\ln(n)^{\ln(n)}}$ converge or diverge?[1]

The remaining pages have some hints. Hold off looking at them until you've thought about these problems for a while. If you cannot state the problem from memory, you are not ready to look at the hints.

[1] In general, $f(x)^a = (f(x))^a$. In particular, $\ln(n)^{\ln(n)} = (\ln(n))^{\ln(n)}$.

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Hints for 1

(a) If you want to use the alternating series test, take special care that you find where $\frac{1}{26-10n+n^2}$ is monotonic.

(b) Really, try graphing it.

Hints for 2

(a) The divergence test is useful for the values of x that other tests might not be able to deal with.

(b) Remember that if $\lim_{n\to\infty} b_n = 2$, then the sequence eventually gets as close to 2 as you might want. For example, there is an N such that $b_n > 3/2$ for all $n \geq N$. (The $3/2$ is just some number between 1 and 2. I chose the average of 1 and 2 for no real reason.)

(c) The quotient $(b_1 b_2 \ldots b_{n+1})/(b_1 b_2 \ldots b_n)$ is b_{n+1}.

Hints for 3

(a) Recall exponentiation rules, especially ones involving putting things in the form $e^{\text{something}}$.

(b) The function $\ln(\ln(n))$ is monotonically increasing without bound.[2]

(c) That means, for any fixed constant, $\ln(\ln(n))$ will eventually be greater than that fixed constant.

(d) You have the liberty to choose a fixed constant that will be useful to you.

(d) Given that idea, try comparing against a p-series.

[2]Hint hint: can you manipulate things so $\ln(\ln(n))$ shows up?