MATH 1B. 10AM 2/14/2020. (15 minutes, 6 points) Name: SOLUTIONS

Quiz 3

1. (3 points) Evaluate the integral $\displaystyle\int \frac{dz}{z+\sqrt{z}}$.

Let $u = \sqrt{z}$. Then $du = \frac{1}{2}z^{-1/2}\,dz$. Solving for dz, this is $dz = 2z^{1/2}\,du = 2u\,du$. Performing the substitution,

$$\begin{aligned}\int \frac{dz}{z+\sqrt{z}} = \int \frac{2u\,du}{u^2+u} &= \int \frac{2\,du}{u+1} \\ &= 2\ln|u+1| + C \\ &= \boxed{2\ln(\sqrt{z}+1)+C}.\end{aligned}$$

2. (3 points) Find a large enough n that guarantees that the trapezoidal rule approximation for $\ln(3) = \int_1^3 t^{-1}\,dt$ is accurate to within 0.01.

The trapezoidal rule requires a bound on the absolute value of the second derivative of the integrand. We can calculate $\frac{d^2}{dt^2}t^{-1} = 2t^{-3}$. On the domain $[1,3]$, the function $2t^{-3}$ is decreasing and positive, so $|2t^{-3}| \le 2(1)^{-3} = 2$ for all $t \in [1,3]$. Hence let $K = 2$.
The trapezoidal rule error bound on $[1,3]$ with this K is

$$|E_T| \le \frac{2(3-1)^3}{12n^2} = \frac{4}{3n^2}.$$

We want this to be less than 0.01, so we have the inequality $\frac{4}{3n^2} < 0.01$. Solving for n, this is

$$\sqrt{\frac{400}{3}} < n.$$

Since $\sqrt{\frac{400}{3}} = 20\sqrt{2/3}$, if we estimate $\sqrt{2/3} < 1$, then we could, if we wanted to, get the concrete estimate of $n \ge 20$, giving an n that's not *too* large that guarantees the trapezoidal rule will work. (In reality, $n > 11.547$, so using $n = 12$ is sufficient.)

Quiz 3

1. (3 points) Evaluate the integral $\displaystyle\int \frac{dy}{\sqrt{y}-y}$.

Let $u=\sqrt{y}$, hence $du = \frac{1}{2}y^{-1/2}\,dy$. Solving for dy, we get $dy = 2y^{1/2}\,du = 2u\,du$. Substituting this in,

$$\int \frac{dy}{\sqrt{y}-y} = \int \frac{2u\,du}{u-u^2} = \int \frac{2\,du}{1-u} = -2\ln|1-u| + C$$
$$= \boxed{-2\ln|1-\sqrt{y}|+C}.$$

2. (3 points) Find a large enough n that guarantees that the midpoint rule approximation for $\ln(3) = \int_1^3 t^{-1}\,dt$ is accurate to within 0.01.

The midpoint rule requires a bound on the absolute value of the second derivative of the integrand. We can calculate $\frac{d^2}{dt^2}t^{-1} = 2t^{-3}$. On the domain $[1,3]$, the function $2t^{-3}$ is decreasing and positive, so $|2t^{-3}| \le 2(1)^{-3} = 2$ for all $t \in [1,3]$. Hence let $K=2$.
The midpoint rule error bound on $[1,3]$ with this K is

$$|E_M| \le \frac{2(3-1)^3}{24n^2} = \frac{2}{3n^2}.$$

We want this to be less than 0.01, so we have the inequality $\frac{2}{3n^2} < 0.01$. Solving for n, this is

$$\sqrt{\frac{200}{3}} < n.$$

Since $\sqrt{\frac{200}{3}} = 10\sqrt{2/3}$, if we estimate $\sqrt{2/3} < 1$, then we could, if we wanted to, get the concrete estimate of $n \ge 10$, giving an n that's not *too* large that guarantees the midpoint rule will work. (In reality, $n > 8.165$, so using $n = 9$ is sufficient.)