MATH 1B. 10AM 1/31/2020. (15 minutes, 9 points) Name: SOLUTIONS

Quiz 1

1. (3 points) Evaluate the integral $\int (x^2+1)\ln(x)\,dx$.

Use integration by parts with

$$f(x) = \ln(x) \qquad g'(x) = x^2 + 1$$
$$f'(x) = x^{-1} \qquad g(x) = \tfrac{1}{3}x^3 + x.$$

Then,

$$\begin{aligned}\int (x^2+1)\ln(x)\,dx &= \ln(x)(\tfrac{1}{3}x^3 + x) - \int x^{-1}(\tfrac{1}{3}x^3 + x)\,dx \\ &= \ln(x)(\tfrac{1}{3}x^3 + x) - \int (\tfrac{1}{3}x^2 + 1)\,dx \\ &= \boxed{\ln(x)(\tfrac{1}{3}x^3 + x) - (\tfrac{1}{9}x^3 + x) + C}.\end{aligned}$$

2. (3 points) Evaluate the integral $\int \sin(\sqrt{t})\,dt$.

First, let $u = \sqrt{t} = t^{1/2}$, where $du = \frac{1}{2}t^{-1/2}\,dt$. Solving for dt, this is $dt = 2t^{1/2}\,du = 2u\,du$, hence

$$\int \sin(\sqrt{t})\,dt = \int \sin(u)\cdot 2u\,du = 2\int u\sin(u)\,du.$$

Using integration by parts with

$$f(u) = u \qquad g'(u) = \sin(u)$$
$$f'(u) = 1 \qquad g(u) = -\cos(u),$$

then

$$\begin{aligned}\int u\sin(u)\,du &= u(-\cos(u)) - \int 1(-\cos(u))\,du \\ &= -u\cos(u) + \int \cos(u)\,du \\ &= -u\cos(u) + \sin(u) + C.\end{aligned}$$

Substituting this in,

$$\begin{aligned}\int \sin(\sqrt{t})\,dt &= 2(-u\cos(u) + \sin(u) + C) \\ &= \boxed{2(-\sqrt{t}\cos(\sqrt{t}) + \sin(\sqrt{t}) + C)}.\end{aligned}$$

3. (3 points) Evaluate the integral $\int \sin(3\theta)\cos(3\theta)\,d\theta$.

There are two main ways to solve this, one easy and unintended, the other using integration by parts.

1. Letting $u = \sin(3\theta)$, we have $du = 3\cos(3\theta)\,d\theta$, so then

$$\int \sin(3\theta)\cos(3\theta)\,d\theta = \int u \cdot \tfrac{1}{3}\,du = \tfrac{1}{6}u^2 + C = \boxed{\tfrac{1}{6}\sin(3\theta)^2 + C}.$$

2. Or we can do integration by parts with

$$\begin{aligned} f(\theta) &= \sin(3\theta) & g'(\theta) &= \cos(3\theta) \\ f'(\theta) &= 3\cos(3\theta) & g(\theta) &= \tfrac{1}{3}\sin(3\theta), \end{aligned}$$

getting

$$\begin{aligned} \int \sin(3\theta)\cos(3\theta)\,d\theta &= \sin(3\theta)(\tfrac{1}{3}\sin(3\theta)) - \int (3\cos(3\theta))(\tfrac{1}{3}\sin(3\theta))\,d\theta \\ &= \tfrac{1}{3}\sin(3\theta)^2 - \int \sin(3\theta)\cos(3\theta)\,d\theta. \end{aligned}$$

Solving for the integral, we obtain

$$\int \sin(3\theta)\cos(3\theta)\,d\theta = \boxed{\tfrac{1}{6}\sin(3\theta)^2 + C}.$$

Quiz 1

1. (3 points) Evaluate the integral $\int (x^2 + 2x)e^x \, dx$.

Use integration by parts with

$$f(x) = x^2 + 2x \qquad g'(x) = e^x$$
$$f'(x) = 2x + 2 \qquad g(x) = e^x.$$

Then,

$$\int (x^2 + 2x)e^x \, dx = (x^2 + 2x)e^x - \int (2x + 2)e^x \, dx.$$

For this new integral, use integration by parts with

$$f(x) = 2x + 2 \qquad g'(x) = e^x$$
$$f'(x) = 2 \qquad g(x) = e^x.$$

Then,

$$\int (2x + 2)e^x \, dx = (2x + 2)e^x - \int 2e^x \, dx = (2x + 2)e^x - 2e^x + C.$$

Substituting this back in,

$$\int (x^2 + 2x)e^x \, dx = \boxed{(x^2 + 2x)e^x - ((2x + 2)e^x - 2e^x + C)}.$$

2. (3 points) Evaluate the integral $\int \cos(\sqrt{t}) \, dt$.

First, let $u = \sqrt{t} = t^{1/2}$, where $du = \frac{1}{2}t^{-1/2} \, dt$. Solving for dt, this is $dt = 2t^{1/2} \, du = 2u \, du$, hence

$$\int \cos(\sqrt{t}) \, dt = \int \cos(u) \cdot 2u \, du = 2 \int u \cos(u) \, du.$$

Using integration by parts with

$$f(u) = u \qquad g'(u) = \cos(u)$$
$$f'(u) = 1 \qquad g(u) = \sin(u),$$

then

$$\int u \cos(u) \, du = u \sin(u) - \int \sin(u) \, du$$
$$= u \sin(u) + \cos(u) + C.$$

Substituting this in,

$$\int \cos(\sqrt{t})\,dt = 2(u\sin(u) + \cos(u) + C)$$
$$= \boxed{2(\sqrt{t}\sin(\sqrt{t}) + \cos(\sqrt{t}) + C)}.$$

3. (3 points) Evaluate the integral $\int \cos(2\theta)\sin(2\theta)\,d\theta$.

There are two main ways to solve this, one easy and unintended, the other using integration by parts.

1. Letting $u = \sin(2\theta)$, we have $du = 2\cos(2\theta)\,d\theta$, so then

$$\int \sin(2\theta)\cos(2\theta)\,d\theta = \int u \cdot \tfrac{1}{2}\,du = \tfrac{1}{4}u^2 + C = \boxed{\tfrac{1}{4}\sin(2\theta)^2 + C}.$$

2. Or we can do integration by parts with

$$f(\theta) = \sin(2\theta) \qquad g'(\theta) = \cos(2\theta)$$
$$f'(\theta) = 2\cos(2\theta) \qquad g(\theta) = \tfrac{1}{2}\sin(2\theta),$$

getting

$$\int \sin(2\theta)\cos(2\theta)\,d\theta = \sin(2\theta)(\tfrac{1}{2}\sin(2\theta)) - \int (2\cos(2\theta))(\tfrac{1}{2}\sin(2\theta))\,d\theta$$
$$= \tfrac{1}{2}\sin(2\theta)^2 - \int \sin(2\theta)\cos(2\theta)\,d\theta.$$

Solving for the integral, we obtain

$$\int \sin(2\theta)\cos(2\theta)\,d\theta = \boxed{\tfrac{1}{4}\sin(2\theta)^2 + C}.$$