Quiz 9

1. (2 points). Find the local maximum and minimum values of $g(x) = 2x^3 + 3x^2 - 12x + 5$ using the Second Derivative Test.

First we will find critical points. Since $g'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1)$, the critical points are at $x = -2, 1$. Second, we compute the second derivative $g''(x) = 12x + 6$.

Because $g''(-2) = -18 < 0$, g has a local maximum at $x = -2$, and because $g''(1) = 18 > 0$, g has a local minumum at $x = 1$.

2. (3 points). Let $f(x) = 1 + \frac{1}{x-2} - \frac{1}{(x-2)^2}$. Find (a) vertical and horizontal asymptotes, (b) intervals of increase or decrease, (c) local maxima and minima, (d) intervals of concavity and inflection points, and (e) x- and y-intercepts.

(a) Since $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 1$, there is a horizontal asymptote $y = 1$ at both ∞ and $-\infty$. The only place f could have a vertical asymptote is at $x = 2$, and we compute $\lim_{x\to 2} f(x) = \lim_{x\to 2}(1 + \frac{x-3}{(x-2)^2}) = -\infty$ since the top of the fraction goes to -1 while the bottom goes to 0^+.

(b) The derivative is $f'(x) = \frac{-1}{(x-2)^2} + \frac{2}{(x-2)^3}$, using the power rule. This is undefined when $x = 2$, but that is outside the domain of f so that does not count as a critical point. It is zero when $0 = -1(x-2) + 2$, which is where $x = 4$. Thus we consider the following intervals:

1. $(-\infty, 2)$, which, since $f'(0) = \frac{-1}{4} + \frac{2}{-8} < 0$, is an interval of decrease.
2. $(2, 4)$, which, since $f'(3) = \frac{-1}{1} + \frac{2}{1} > 0$, is an interval of increase.
3. $(4, \infty)$, which, since $f'(5) = \frac{-1}{9} + \frac{2}{27} < 0$, is an interval of decrease.

(c) There is a local maximum at $x = 4$.

(d) The second derivative is $f''(x) = \frac{2}{(x-2)^3} - \frac{6}{(x-2)^4}$, which again has the same domain as f. It is zero when $0 = 2(x-2) - 6$, which is when $x = 5$. We consider the following intervals:

1. $(-\infty, 2)$, which since $f''(1) = \frac{2}{-1} - \frac{6}{1} < 0$ means the interval is concave down.
2. $(2, 5)$, which since $f''(3) = \frac{2}{1} - \frac{6}{1} < 0$ means the interval is concave down.
3. $(5, \infty)$, which since $f''(6) = \frac{2}{4^3} - \frac{6}{4^4} = \frac{8-6}{4^4} > 0$ means the interval is concave up.

As the concavity changes at $x = 5$, this is an inflection point.

(e) The y-intercept is $f(0) = 1 + \frac{1}{-2} - \frac{1}{4} = \frac{1}{4}$. The x-intercept is when $(x-2)^2 + (x-2) - 1 = 0$, and so by the quadratic equation, $x - 2 = \frac{-1 \pm \sqrt{1+4}}{2}$, which is $x = \frac{3 \pm \sqrt{5}}{2}$.

You can make a pretty nice graph using all of these features. Make sure and compute $f(4)$ and $f(5)$ to plot the critical point and the inflection point properly. (Check the graph on WolframAlpha or something.)