Quiz 7

Please note this quiz has two sides! .

1. (2 points). Find the derivative of the function. Simplify your answer.

$$y = \sqrt{e^{\sin x}}$$

Solution:

$$\begin{aligned} y &= \sqrt{e^{\sin x}} \\ &= (e^{\sin x})^{\frac{1}{2}} \\ y' &= \frac{1}{2}(e^{\sin x})^{-\frac{1}{2}} \times (e^{\sin x})' \\ &= \frac{1}{2}(e^{\sin x})^{-\frac{1}{2}} \times (e^{\sin x}) \times \cos x \\ &= \frac{1}{2}(e^{\sin x})^{\frac{1}{2}} \times \cos x \\ &= \frac{1}{2}\sqrt{e^{\sin x}} \times \cos x \end{aligned}$$

Here we apply the chain rule twice.

2a. (1 point). Compute the derivative of $\tan(x)$. You must show your work; just writing down the answer will receive no credit.

Solution:

$$\begin{aligned}\frac{d}{dx}\tan(x) &= \frac{d}{dx}\frac{\sin(x)}{\cos(x)} \\ &= \frac{\cos(x)\sin(x)' - \cos(x)'\sin(x)}{\cos^2(x)} \\ &= \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} \\ &= \frac{1}{\cos^2(x)} = \boxed{\sec^2(x)}\end{aligned}$$

2b. (2 points). Find $\frac{dy}{dx}$ by implicit differentiation. Simplify your answer.

$$xy = \tan(xy)$$

Solution:

Taking the derivative with respect to x on both sides:

$$\frac{d}{dx}xy = \frac{d}{dx}\tan(xy)$$

Using the product rule and the chain rule on the left hand side (remembering that y is a function of x):

$$y + x\frac{dy}{dx} = \frac{d}{dx}\tan(xy)$$

Using the fact from 2a that $\frac{d}{dx}\tan(x) = \sec^2(x)$ and the chain rule on the right hand side:

$$y + x\frac{dy}{dx} = \sec^2(xy) \times \frac{d}{dx}(xy)$$

$$y + x\frac{dy}{dx} = \sec^2(xy)(y + x\frac{dy}{dx})$$

Solving for $\frac{dy}{dx}$:

$$y + x\frac{dy}{dx} = \sec^2(xy)y + \sec^2(xy)x\frac{dy}{dx}$$

$$x\frac{dy}{dx} - \sec^2(xy)x\frac{dy}{dx} = \sec^2(xy)y - y$$

$$\frac{dy}{dx}x(1-\sec^2(xy)) = -y(1-\sec^2(xy))$$

$$\boxed{\frac{dy}{dx} = -\frac{y}{x}}$$