You have 20 minutes to complete the quiz. No calculators.

Name:____

1. (2 points) Write down the equation for the line tangent to the curve $f(x) = 3x^2 - 2x$ at $x = 1$.

Solution.

$$f'(1) = \lim_{x\to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1} \frac{3x^2 - 2x - 1}{x - 1} = \lim_{x\to 1} \frac{(3x+1)(x-1)}{x-1} = \lim_{x\to 1}[3x+1] = 4.$$

Furthermore, $f(1) = 1$, and hence the equation of the line is

$$y = 4(x - 1) + 1.$$

□

2. (3 points) Define

$$f(x) := \frac{3x^3 - 2x^2 - 5}{x^3 - 8}.$$

(a) Using the fact that $\lim_{x\to\infty} \frac{1}{x^n} = 0$ for $n > 0$, compute $\lim_{x\to\infty} f(x)$.

Solution.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3 - \frac{2}{x} - \frac{5}{x^3}}{1 - \frac{8}{x^3}} = 3.$$

□

(b) What are the horizontal and vertical asymptotes of f?

Solution. By part (a), it has horizontal asymptotes as $x \to \pm\infty$ at $y = 3$. The denominator has a root at $x = 2$ (and furthermore, this is the denominator's only real root), whereas the numerator does not. Therefore, it has a single vertical asymptote at $x = 2$. □

(c) Recall the definition for limits as $x \to \infty$: We say that $\lim_{x\to\infty} f(x) = L$ if and only if for every $\epsilon > 0$ there is some $M \geq 0$ such that whenever $x > M$ it follows that $|f(x) - L| < \epsilon$. Using the definition, prove that $\lim_{x\to\infty} \frac{1}{x^n} = 0$ for n a positive integer (i.e. n can be $1, 2, \ldots$). Start by finding M for a given ϵ and n.

Solution. Let $\epsilon > 0$. Define $M := \frac{1}{\epsilon^{1/n}}$. Suppose that $x > M$. Then,

$$\left|\frac{1}{x^n} - 0\right| = \frac{1}{x^n} < \frac{1}{M^n} = \epsilon. \tag{1}$$

□