MATH 1A — Calculus Quiz 4 25 February 2015

You have 20 minutes to complete the quiz. No calculators.

Name:_____

1. (5 points) Let $f(x) : [0,1] \to [0,1]$ be continuous. Show that there is some $x_0 \in [0,1]$ such that $f(x_0) = x_0$. We will guide you through the argument. If by chance you find the guidance more confusing than helpful, feel free to write your own proof from scratch.

First, we define $g(x) := f(x) - x$.

(a) Explain why $g(x_0) = 0$ is equivalent to $f(x_0) = x_0$. Thus, instead of solving the original problem, we will find an $x_0 \in [0,1]$ such that $g(x_0) = 0$.

(b) Either $g(0) = 0$, $g(0) < 0$, or $g(0) > 0$. If $g(0) = 0$, we are done. Why? What is the x_0 that works?

(c) In fact, it cannot be the case that $g(0) < 0$. Why? Thus, unless we have found an x_0 that works in part (c), we know that it must be the case that $g(0) > 0$.

(d) Go through a similar argument with $g(1)$: if $g(1) = 0$ we are done; if not, what is the sign of $g(1)$?

(e) At this point, we know (i) that g is continuous[1] and (unless we have already found an x_0 in parts (b) or (d)) (ii) the sign of $g(0)$ and the sign of $g(1)$. Apply the Intermediate Value Theorem to deduce that there is some point $x_0 \in [0,1]$ such that $g(x_0) = 0$.

[1] There are not enough points to grade you on this, but if you have time left over, you should think real quick about why we know g is continuous.