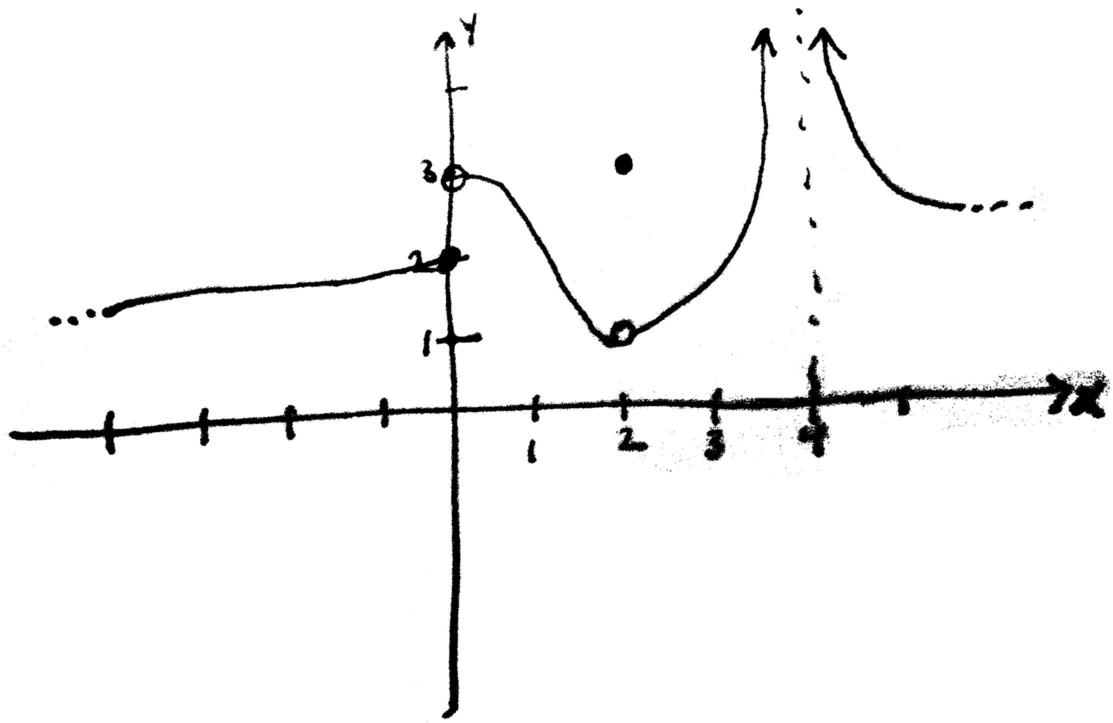
MATH 1A. February 11, 2015. 9am. (20 minutes) Name:

Quiz 3

1. (2 points). Sketch the graph of a function $f(x)$ that satisfies the conditions

$$\lim_{x\to 0^-} f(x) = 2, \quad \lim_{x\to 0^+} f(x) = 3, \quad \lim_{x\to 2} f(x) = 1, \quad f(2) = 3, \quad \lim_{x\to 4} f(x) = \infty$$

Answers vary, but here is a possible example:



Important points: at $x = 2$ there must be an open and closed point, at $x = 4$ it must be a two-sided limit to ∞. At $x = 0$, either or neither of $y = 2, 3$ might be filled in.

2. (3 points). Evaluate the following two limits. (No L'Hospital's Rule!) Show all work.

a. $\displaystyle\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 4}$ b. $\displaystyle\lim_{x\to 0^+} x^3 \cos(\ln x)$

a.

$$\begin{aligned}\lim_{x\to 2} \frac{x^3-8}{x^2-4} &= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} \\ &= \lim_{x\to 2} \frac{x^2+2x+4}{x+2} && \text{since } x \neq 2 \\ &= \frac{2^2 + 2\cdot 2 + 4}{2+2} \\ &= 3.\end{aligned}$$

b. First notice that $\ln x$ does not matter here, other than checking that $\ln x$ is defined for $x > 0$. Then, from knowing the range of cosine we have $-1 \leq \cos(\ln x) \leq 1$, so, since $x^3 > 0$ for $x > 0$, we have $-x^3 \leq x^3 \cos(\ln x) \leq x^3$, and since $\lim_{x \to 0^+} \pm x^3 = 0$ (direct substitution), by the squeeze theorem we obtain

$$\lim_{x \to 0^+} x^3 \cos(\ln x) = 0.$$