Quiz 1

1. (2 points). Define $f(x) = x^2 + 3$. Expand and simplify

$$\frac{f(x+h) - f(x)}{h}.$$

To expand the expression, we substitute $f(x+h) = (x+h)^2 + 3$ and $f(x) = x^2 + 3$:

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{((x+h)^2 + 3) - (x^2 + 3)}{h} \\
&= \frac{(x+h)^2 - x^2}{h} \\
&= \frac{x^2 + 2xh + h^2 - x^2}{h} \\
&= \frac{2xh + h^2}{h} \\
&= 2x + h.
\end{aligned}$$

2. (3 points). Define $f(x) = \frac{x^3+8}{x+2}$ and $g(x) = x^2 - 2x + 4$. What are the domains of both f and g, respectively? Are f and g the same function? Why or why not?

The domain of f is $(-\infty, -2) \cup (-2, \infty)$ (also written as $x \neq -2$ or $\{x \in \mathbb{R} : x \neq -2\}$) because there is a division by zero when $x = -2$. For g, the domain is all of $\mathbb{R}$. Since the domains of f and g are different, they are different functions.

(However: notice that $x^3 + 8 = (x+2)(x^2 - 2x + 4)$, so when $x \neq -2$, $f(x) = g(x)$. This means that *when we restrict their domains to* $x \neq -2$, f and g are the same function.)