MATH 1A — Calculus Quiz 10 22 April 2015

You have 20 minutes to complete the quiz. No calculators.

Name:_____

1. (2 points) Evaluate $\lim_{x\to 0}(x+1)^{1/\sin x}$.

Solution.

$$\begin{aligned}\lim_{x\to 0}(x+1)^{1/\sin x} &= \lim_{x\to 0} e^{\ln(x+1)^{1/\sin x}} \\ &= e^{\lim_{x\to 0}\frac{ln(x+1)}{\sin x}} \\ &= e^{\lim_{x\to 0}\frac{1/(x+1)}{\cos x}} \\ &= e^{\frac{1/(0+1)}{\cos 0}} \\ &= e.\end{aligned}$$

1/2 point for using logarithms, 1 point for using L'Hospital's rule correctly, 1/2 point for getting e. □

2. (3 points) Graph $y = \frac{4}{x^2+3}$. To do this, (a) find the intervals of increase/decrease, (b) find the local extrema, (c) find the intervals where the graph is concave up/down, and (d) find the inflection points. Find the y-coordinate for local extrema and inflection points.

Solution. $y' = -\frac{2x}{(x^2+3)^2}$ and $y'' = \frac{6(x^2-1)}{(x^2+3)^3}$. Increases on $(-\infty, 0)$ and decreases on $(0, \infty)$; local (absolute) maximum at $(0, 4)$; concave up on $(-\infty, -1)$ and $(1, \infty)$; concave down on $(-1, 1)$; inflection points at $(-1, 1)$ and $(1, 1)$. Half point for finding y', half point for finding y'', half point for finding intervals of increase/decrease and local extrema, half point for finding intervals of concavity and inflection points; one point for drawing a reasonable graph. □