Review for Midterm 2

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Make sure to study the topics in the midterm 1 review as well. For instance, make sure you are comfortable with the rules of logarithms.

1. Be able to define (*precisely*): continuity of a function, the derivative of a function, differentiability of a function. Know that differentiable imples continuous but not vice versa. Know examples of not being continuous and of not being differentiable.

2. Be comfortable with piecewise functions with respect to continuity and differentiability.

3. Be able to define, with limits, what it means for a function to have asymptotes (vertical and horizontal). Be able to find the vertical and horizontal asymptotes of rational functions. Be aware of removable singularities.

4. The formula for the tangent line of a differentiable function at a point.

5. Differentiation rules (where full understanding is knowing how to derive, at least intuitively, and how to apply). For instance (with c a constant and $f' = \frac{df}{dx}$): $c' = 0$, $x' = 1$, $(x^c)' = cx^{c-1}$, $(cf)' = cf'$, $(f+g)' = f' + g'$, $(fg)' = fg' + f'g$ [the "product rule"], $(c^x)' = \ln(c)c^x$, $(1/f)' = -f'/f^2$, $(f/g)' = (gf' - fg')/g^2$ [the "quotient rule"], the derivatives of trigonometric functions, the derivatives of inverse trigonometric functions (implicit differentiation can help mnemonically), $(f^{-1})' = 1/f'(f^{-1}(x))$, $(f(g(x)))' = f'(g(x))g'(x)$ [the "chain rule"], $(\ln x)' = 1/x$, $(e^x)' = e^x$.

6. Being able to use implicit differentiation to find tangent lines of plane curves (for instance of $x^2 + xy + y^2 = 2$ or $x^3 + y^3 = 6xy$, at least for horizontal and vertical tangent lines). Do not forget the chain rule, and be comfortable taking the derivative with respect to variables other than x.

7. Logarithmic differentiation. For instance the derivative $(f^g)' = gf^{g-1}f' + \ln(f)f^g g'$ for functions f and g.

8. Linear approximation of a function at a point (i.e., a tangent line). The terse form of the formula is $\Delta y \approx f'(x)\Delta x$.

9. Linear approximations of $\sin x$, $\cos x$, e^x, and $\ln(1+x)$ at $x = 0$.

10. The interpretation of differentials (dy) as errors (Δy).

11. The (*precise*) definitions of absolute minima and maxima. Of local minima and maxima.

12. The Intermediate Value Theorem. Being able to show the existence of roots (i.e., zeros) for odd degree polynomials.

13. The Extreme Value Theorem. Fermat's Theorem. Critical numbers and their relationship to finding extrema (every global extremum is a critical number). Draw a Venn diagram to sort out the following dichotomies: 1) f' exists at c versus f' does not exist at c; 2) f

has a local extremum at c versus f does not; 3) if f' exists at c, $f'(c) = 0$ versus $f'(c) \neq 0$. Come up with examples for each condition as well as a counterexample to the Extreme Value Theorem when the domain is not a closed interval.

14. Optimization problems. The goal is to find global extrema (with the help of Fermat's theorem if the derivative exists). Do not forget that endpoints are critical numbers. If endpoints do not exist (e.g., the domain is not a closed interval), then there might be no global extrema since we may no longer rely on the Extremal Value Theorem. Come up with an example of this.

15. General problem Solving Strategy: 1. Read the problem carefully. 2. Draw a diagram. 3. Introduce notation, labeling knowns and unknowns. 4. Write equation relating variables. 5. Find derivatives as needed. 6. Solve for unknowns. Basically, "What is unknown? How can I relate it to something that I can come to know?" and "Keep track of what I know and do not know."

16. Things that are good to be familiar with: the law of cosines; the Pythagorean theorem; the volume and surface area of spheres, rectangular prisms, cylinders, and cones; the area of rectangles and triangles; the relationship between arc length and subtended angle.

17. The Mean Value Theorem. Prove that a function f is a constant function if and only if $f'(x) = 0$ for all x. Prove that a function is increasing if (and only if) the derivative is always positive.

18. What "if and only if" means. (Maybe look up "contrapositive" and "converse.")

19. l'Hospital's rule (for $0/0$ and ∞/∞), and how to get various indeterminate forms to surrender themselves to the rule of l'Hospital. In particular, $0 \cdot \infty$, $\infty - \infty$, 1^∞, 0^0, ∞^0. Devise examples of indeterminate forms which limit to any value of your choosing.

20. Know the sine and cosine of $0, \pi/6, \pi/4, \pi/3, \pi/2$, know $\sqrt{2}, \sqrt{2}/2, \sqrt{3}/2, \pi, e$ to a few decimal places.

21. For each of these, come up with a problem yourself to solve.