MATH 1A: Spring 2015 **Midterm 2**

Name: _____

Student ID: _____

GSI: _____

The exam is closed book, apart from a sheet of notes 8"x11". Calculators and smart-phones are not allowed. For full credit, you need to show all the reasoning that goes into solving the problem, step by step – the answer alone is not enough. It is your responsibility to write your answers clearly.

There are *two pages* of problems. Please write solutions in blue books.

Problem 1 _____

Problem 2 _____

Problem 3 _____

Problem 4 _____

Problem 5 _____

Problem 6 _____

Problem 7 _____

Total _____ *out of 55*

1. (8 points) *Let*

$$f(x) = xe^x$$

(a) Find the first derivative of $f(x)$.

(b) Find the second derivative of $f(x)$

(c) What is the 17-th derivative of $f(x)$?

2. (9 points) *Let*

$$g(x) = \sqrt{1 - \frac{1}{e^{3x} + 1}}.$$

(a) Find the derivative of $g(x)$.

(b) Find the linear approximation to $g(x)$ at $x = 0$

(c) Estimate $g(0.1)$ and write the answer keeping one decimal place.

3. (7 points) *Find the derivative of*

$$g(x) = (x^2 + 1)^{(x^2+1)}$$

4. (8 points) *Find the points on the curve*

$$x^2 + 2x^4 + 3y^2 = 5$$

where the tangent is horizontal.

5. (6 points) *Let*

$$h(t) = \begin{cases} 3t^2, & t < 1 \\ at^2 + bt + c, & 1 \leq t \end{cases}$$

For which value of the constants a, b and c is the function $h(t)$ continuous and differentiable and second-differentiable for all t?

6. (8 points) *Let*

$$f(x) = \frac{x}{x^2 + 4}$$

(a) On an interval from $[-10, 10]$, find all the critical points of $f(x)$.

(b) Find the absolute maxima and minima of the function on the interval in $[-10, 10]$.

(c) Prove that the function has no local extrema on $[10, \infty)$ (recall that the endpoints of the interval do not count for the purpose).

7. (9 points)

(a) Suppose f is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable. Use implicit differentiation to find the derivative of f^{-1}. Show that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

provided the denominator is not 0.

(b) If $f(3) = 4$ and $f'(3) = \frac{2}{3}$, find $(f^{-1})'(4)$.

(c) Derive the formula for the derivative of

$$g(x) = arc\csc x.$$

Recall that $\csc y = \frac{1}{\sin y}$.