

Name: _____

Student ID: _____

GSI: _____

The exam is closed book. Calculators and smart-phones, or any other digital devices capable of computation are not allowed. For full credit, you need to show all the reasoning that goes into solving the problem, step by step – the answer alone is not enough. It is your responsibility to make your answers clear.

There are *two pages* of problems. Please write solutions in blue books.

Problem 1 _____

Problem 2 _____

Problem 3 _____

Problem 4 _____

Problem 5 _____

Problem 6 _____

Total _____ *out of 30*

1. (4 points) *Let*

$$f(x) = \frac{(x^2 + 1)(x - 1)^2}{(x^2 - 1)^2}$$

Find the following limits (possibly as infinite limits). Justify your answers.

(a)

$$\lim_{x \rightarrow 0} f(x)$$

(b)

$$\lim_{x \rightarrow 1} f(x)$$

(c)

$$\lim_{x \rightarrow -1} f(x)$$

(d) Is $f(x)$ continuous for all x ? Find the points of discontinuity, if any.

2. (6 points) *Let*

$$f(x) = 1 + \sqrt{x - 1}, \quad g(x) = \ln(2x - 1).$$

(a) Find the domain of the function $f(x)$.

(b) Find the domain of the function $g(x)$.

(c) Find $g \circ f(x)$ and its domain.

(d) Find $f \circ g(x)$ and its domain.

3. (5 points) *Let*

$$f(x) = \sqrt{1 - \frac{1}{e^x + 1}}.$$

Find the inverse of $f(x)$. Is 7 in the range of $f(x)$?

4. (5 points) *Let*

$$f(x) = x^4 \sin\left(\frac{5}{x}\right).$$

Use the squeeze theorem to find the following limit.

$$\lim_{x \rightarrow 0} f(x)$$

5. (5 points) *Let*

$$h(t) = \begin{cases} a, & t < 2 \\ 1 + at, & 2 \leq t \end{cases}$$

- (a) For which value of the constant a is the function $h(t)$ continuous for all t ?
- (b) Write down the function $g(t)$ related to $h(t)$ by shifting the graph of $h(t)$ one unit to the left.
- (c) Having found a , sketch the graph of $h(t)$ and $g(t)$. (Your sketch should be at least qualitatively correct.)

6. (5 points) *Recall the definition of*

$$\lim_{x \rightarrow 2} (2x + 3) = 7$$

- (a) For every $\epsilon > 0$ there exists $\delta > 0$ such that

$$\text{if } |x - A| < B, \quad \text{then } |2x + 3 - C| < D$$

Find the missing numbers A, B, C and D in the definition.

- (b) For a given $\epsilon > 0$, find a $\delta > 0$ for which the statement holds. If $\epsilon = 1/10$, what is δ ?