Review for Math 1A Final

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Make sure to study the topics in "Midterm 1 review" and "Midterm 2 review" as well.

1. Be able te define what it means for a function to have asymptotes (vertical, horizontal, and slant).

2. The Mean Value Theorem. Prove that a function f is a constant function if and only if $f'(x) = 0$ for all x. Prove that a differentiable function is increasing if (and only if) the derivative is always positive. Prove that $x + 1 \le e^x$ for all $x \ge 0$.

3. l'Hospital's rule (for $0/0$ and ∞/∞), and how to get various indeterminate forms to surrender themselves to the rule of l'Hospital. In particular, $0 \cdot \infty$, $\infty - \infty$, 1^∞, 0^0, ∞^0. Devise examples of indeterminate forms which limit to any value of your choosing.

4. The increasing/decreasing test. The first derivative test. The definition of concave upward/downward. The concavity test. Inflection points. The second derivative test (positive is happy face, so local minimum; negative is sad face, so local maximum).

5. Curve sketching guidelines: A. Domain. B. Intercepts. C. Symmetry. D. Asymptotes. E. Increasing/Decreasing intervals. F. Local optima. G. Concavity and points of inflection. Remember for each interesting point to compute $f(x)$ and $f'(x)$ there (if it exists) to enhance the sketch.

6. Define the antiderivative of a function. Prove that the difference between two antiderivatives on an interval is a constant (hence, if $F(x)$ is an antiderivative of f, then every antiderivative is of the form $F(x) + C$ with C a constant).

7. Remember an antiderivative for each of the following functions of x, where F, G are antiderivatives of f, g, respectively: x^n ($n \neq -1$), $1/x$, $cf(x)$, $f(x) + g(x)$, e^x, $\cos x$, $\sin x$, $\sec^2 x$, $\sec x \tan x$, $1/\sqrt{1 - x^2}$, $1/(1 + x^2)$.

8. Write down the definition of the Riemann definite integral $\int_a^b f(t)\,dt$ of a continuous function f from a to b. Interpret the parts of the definition intuitively as areas of rectangles. Be aware of left-, right-, and mid- Riemann integral formulations (though in the limit they give the same result).

9. Find the area between $y = \sin x$ and $y = 0$ on $[0, 2\pi]$. Make sure the areas are *positive.*

10. If you can stomache it, compute the definite integral $\int_0^3 (x^2 + x + 1)\,dx$ from the definition of the definite integral. You will need the sum formulas 5,6,7 on page 374.

11. Remember the following properties: $\int_a^b dx = b - a$, $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$, $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$. Interpret these geometrically.

12. Consider the following comparison test: if $f(x) \leq g(x)$ and $a \leq b$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$. Interpret this geometrically. What does this mean if f or g is constant 0? What does this mean if f or g is a constant? (These are some useful consequences.)

13. Fundamental Theorem of Calculus 1. If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of f. (In other words, antiderivatives always exist, and you can get at least one from the definite integral.)

14. Fundamental Theorem of Calculus 2. If f is continuous on $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$. (In other words, any antiderivative will do to compute a definite integral.) We sometimes write $F(b) - F(a)$ as $[F(x)]_a^b$ or just $F(x)]_a^b$.

15. Come up with and solve a problem involving the derivative of a function of x defined by a definite integral whose bounds are themselves functions of x. (In general, if $y = \int_{g(x)}^{h(x)} f(t)\,dt$ and if F is some arbitrary antiderivative of f, then $y = F(h(x)) - F(g(x))$, so $y' = f(h(x))h'(x) - f(g(x))g'(x)$, by the chain rule.)

16. Because of FTC1, we write the "indefinite integral" $\int f(x)\,dx$ for the antiderivatives of f. Why is "antiderivatives" plural? How do you account for it when computing an indefinite integral?

17. Net change theorem. $f(b) = f(a) + \int_a^b f'(x)\,dx$. Why is this true? Come up with physical examples of this theorem.

18. The substitution rule ("u-substitution"). If $u = g(x)$ is a differentiable function, then $\int f(g(x))g'(x)\,dx = \int f(u)\,du$. If g is one-to-one, then $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$. In practice, this means computing the implicit derivative $du = g'(x)\,dx$ and getting rid of x and dx by replacing dx with $du/g'(x)$ and replacing all resulting $g(x)$ with u. A trickier example is $\int_1^2 t^3\sqrt{t^2 - 1}\,dt$ (hint: t^3 is $t \cdot t^2$). A less tricky example is $\int \tan x\,dx$ (hint: tan is a quotient).

19. Integrals $\int_{-a}^{a} f(t)\,dt$ of odd or even continuous f.

20. If $f(x) \geq g(x)$ for all x in $[a, b]$, what is the area between the curves on that interval as an integral? What if f and g cross?