Quiz 1

1. (5 points). Solve the equation $(1-z)+6(1-z)^{-1}=5$ for z.

   We can solve this in two different ways. Both of them start by multiplying each side by $1-z$, to get, using the fact that $(1-z)^{-1}\cdot(1-z)=1$,

   $$(1-z)^2+6=5(1-z).$$

   And then we subtract $5(1-z)$ from each side to get

   $$(1-z)^2-5(1-z)+6=0.$$

   Now, the first way to solve this is to realize that this the quadratic $w^2-5w+6=0$, where $w=1-z$. Solving for $1-z$ with the quadratic formula,

   $$\begin{aligned} 1-z &= \frac{5\pm\sqrt{(-5)^2-4\cdot 1\cdot 6}}{2} \\ &= \frac{5\pm 1}{2} \\ &= 2,3. \end{aligned}$$

   And finally, solving for z, we have $\boxed{z=-2,-1}$.

   Rather than being clever, we can also multiply out the $(1-z)^2$ and get

   $$\begin{aligned} 0 &= 1-2z+z^2-5+5z+6 \\ 0 &= z^2+3z+2, \end{aligned}$$

   with which one could either factor or use the quadratic formula to obtain the solutions.

   Just for our sanity, we could check that $1-z\neq 0$ each of these z, since the given equation is not defined in that case. We have $1-(-1)=2\neq 0$ and $1-(-2)=3\neq 0$, so neither are "fake" solutions.

2. (5 points). Simplify $\frac{x^2-x-3}{x+3}+\frac{3x}{x-3}-\frac{18x}{x^2-9}$. For which x is this defined?

   Notice that $x^2-9=(x-3)(x+3)$, and so the common denominator we need to combine the fractions is just x^2-9. Thus, we multiply the first fraction by $\frac{x-3}{x-3}$ and the second by $\frac{x+3}{x+3}$:

$$\begin{aligned}
&= \frac{x^2-x-3}{x+3}\cdot\frac{x-3}{x-3}+\frac{3x}{x-3}\cdot\frac{x+3}{x+3}-\frac{18x}{(x+3)(x-3)}\\
&= \frac{(x^2-x-3)(x-3)+3x(x+3)-18x}{(x+3)(x-3)}\\
&= \frac{x^3-x^2-3x-3x^2+3x+9+3x^2+9x-18x}{(x+3)(x-3)}\\
&= \frac{x^3-x^2-9x+9}{(x+3)(x-3)}\\
&= \frac{x(x^2-9)-(x^2-9)}{x^2-9}\\
&= \frac{x(x^2-9)}{x^2-9}-\frac{x^2-9}{x^2-9}\\
&= \boxed{x-1}.
\end{aligned}$$

The algebra is simpler if one notices that the first fraction $\frac{x^2-x-3}{x+3}$ is $\frac{x^2-(x+3)}{x+3}=\frac{x^2}{x+3}-1$. Also, because x^2-9 is in the denominator (which has roots at $x=\pm 3$), the expression is defined whenever $\boxed{x\neq\pm 3}$.

3. (5 points). At which points (x, y) do the lines $5x-2y=6$ and $3x+y=8$ intersect?

   Solving the second equation for y, we get $y=8-3x$, which we can substitute into the first equation to get $5x-2(8-3x)=6$. Simplifying, this equation becomes $11x-16=6$, and solving for x we have $x=2$. Substituting this into $y=8-3x$ we have $y=8-3\cdot 2=2$. Therefore, the lines intersect at $\boxed{(2,2)}$.