Review for Midterm 2

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Kyle Miller

Make sure to study the topics in "One-Minute Quickes for Midterm 1" as well.

1. Be able to define (*precisely*): continuity of a function, the derivative of a function, differentiability of a function. Know that differentiable imples continuous but not vice versa. Know examples of not being continuous and not being differentiable.

2. Be able te define what it means for a function to have asymptotes (vertical, horizontal, and slant).

3. The formula for the tangent line of a differentiable function at a point.

4. Differentiation rules (where full understanding is knowing how to derive, at least intuitively, and to apply). For instance (with c a constant and $f' = \frac{df}{dx}$): $c' = 0$, $x' = 1$, $(x^c)' = cx^{c-1}$, $(cf)' = cf'$, $(f + g)' = f' + g'$, $(fg)' = fg' + f'g$ [the "product rule"], $(e^x)' = e^x$, $(1/f)' = -f'/f^2$, $(f/g)' = (gf' - fg')/g^2$ [the "quotient rule"], the derivatives of trigonometric functions, the derivatives of inverse trigonometric functions (implicit differentiation can help as a mnemonic), $(f^{-1})' = 1/f'(f^{-1}(x))$, $(f(g(x)))' = f'(g(x))g'(x)$ [the "chain rule"], $(c^x)' = \ln(c)c^x$, $(\ln x)' = 1/x$.

5. Being able to do implicit differentiation to find tangent lines of plane curves (for instance of $x^2 + xy + y^2 = 2$ or $x^3 + y^3 = 6xy$, at least for horizontal and vertical tangent lines).

6. Logarithmic differentiation. The derivative $(f^g)' = gf^{g-1}f' + f^g \ln(f)g'$ for functions f and g (notice it is both the power and exponentiation rules added together).

7. Be familiar with df/dt as a rate of change (see Sections 3.7-3.9). Practice related rates problems. Problem Solving Strategy: 1. Read the problem carefully. 2. Draw a diagram. 3. Introduce notation, labeling knowns and unknowns. 4. Write equation relating variables. 5. Find derivatives as needed. 6. Solve for unknowns. Basically, "What is unknown? How can I relate it to something that I can come to know?" and "Keep track of what I know and do not know."

8. The law of cosines; the Pythagorean theorem; the volume and surface area of spheres, rectangular prisms, cylinders, and cones; the area of rectangles and triangles; the relationship between arc length and subtended angle.

9. Linear approximations (i.e., tangent lines) of $\sin x$, $\cos x$, and e^x at $x = 0$.

10. The interpretation of differentials as errors.

11. The (*precise*) definitions of absolute minima and maxima. Of local minima and maxima.

12. The Intermediate Value Theorem. Being able to show the existence of roots (i.e., zeros) for odd degree polynomials.

13. The Extreme Value Theorem. Fermat's Theorem. Critical numbers and their relationship to finding optima.

14. The Mean Value Theorem. Prove that a function f is a constant function if and only if $f'(x) = 0$ for all x. Prove that a function is increasing if (and only if) the derivative is always positive.

15. What "if and only if" means. The contrapositive and converse (and that the first is true and the second is not in general; think of examples).

16. The increasing/decreasing test. The first derivative test. The definition of concave upward/downward. The concavity test. Inflection points. The second derivative test.

17. l'Hospital's rule (for $0/0$ and ∞/∞), and how to get various indeterminate forms to surrender themselves to the rule of l'Hospital. In particular, $0 \cdot \infty$, $\infty - \infty$, 1^∞, 0^0, ∞^0.

18. Curve sketching guidelines: A. Domain. B. Intercepts. C. Symmetry. D. Asymptotes. E. Increasing/Decreasing intervals. F. Local optima. G. Concavity and points of inflection. Remember for each interesting point to compute $f(x)$ and $f'(x)$ there (if it exists) to enhance the sketch.

19. Optimization problems. Essentially the same problem solving strategy as for related rates, however the goal is to find global optima (usually with the help of Fermat's theorem). Do not forget endpoints. If endpoints do not exist, then check the limits.

20. For each of these, come up with a problem yourself to solve.