One-Minute Quickies for Midterm 1

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Kyle Miller

1. Define *function*, *domain*, *target*, *range*.
2. Draw graphs for $f(x) = x$, x^2, x^3, $\sqrt{x}$, $\sqrt[3]{x}$, $\frac{1}{x}$, $\frac{1}{x^2}$, $\sin x$, $\cos x$, $\tan x$, $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, e^x, $\ln x$, $|x|$. What are their domains? Their ranges?
3. Give a precise definition for a function to be *even*, *odd*, *increasing*, *decreasing*, *injective* (*one-to-one*), *surjective*, and *bijective*. Find an example for each of these.
4. Define the *inverse* of a function. When does an inverse exist?
5. Find an example of functions f and g where $g \circ f$ is bijective but neither of f nor g are.
6. Suppose f is a function. What are the graphs of $g_1(x) = f(x - a)$, $g_2(x) = f(x) + b$, $g_3(x) = cf(x)$, and $g_3(x) = f(dx)$ relative to the graph of f? Consider all possible values of a, b, c, d.
7. What is the function for a line which goes through points (x_1, y_1) and (x_2, y_2)?
8. What is a root of a polynomial? What are the roots of a quadratic?
9. What is a rational function? When do *asymptotes* or *removable singularities* happen for these? (How does one find them? Examples?)
10. What are the rules for multiplying exponentials, adding/subtracting logarithms, and changing the bases for exponentials and logarithms?
11. What is the formal definition of $\lim_{x \to a} f(x) = L$? An informal definition?
12. What does it mean for f to be continuous at a point a?
13. What does it mean for f to be continuous on an interval? What is an informal definition? Prove linear functions in general are continuous.
14. Give main examples of kinds of discontinuities as well as functions possessing them.
15. What are the definitions of left- and right-sided limits? In terms of these, when does a limit exist?
16. What does it mean if $\lim_{x \to a} f(x) = \infty$? What does it mean if $\lim_{x \to \infty} f(x) = L$? If $\lim_{x \to \infty} f(x) = \infty$? Define *vertical asymptote* and *horizontal asymptote*.
17. What are the limit laws (addition, multiplication, composition, etc.)?
18. State the squeeze (sandwich) theorem and the intermediate value theorem.
19. What is the conjugate of a rational expression? Use this to compute some limit.
20. Define the derivative (slope) of a function f at a point a. Compute the derivative of x^2.

Selected problems from previous midterms

- $\lim_{x\to 1} \dfrac{x^2+7x-8}{x^3-1}$
- $\lim_{x\to -2^+} \dfrac{1}{x^2-4}$
- $\lim_{x\to\infty} \tan^{-1}\left(\dfrac{x^2+2x+7}{x^2+6x}\right)$
- $\lim_{x\to\infty} (\sqrt{x^2+1}-x)$
- A formula for the inverse of $f(x)=\sqrt{2+3x}$, state its domain and range (be careful).
- $\lim_{x\to 0^+} x\tan(\sin(\ln x))$
- Vertical and horizontal asymptotes of $\frac{\sqrt{x^6+4x^5+3x}-x^3}{x^2-x}$.
- $\lim_{x\to 3^-} \dfrac{\ln x}{x-3}$
- Graph $f(x)=|x^2+2x|$.
- Graph $g(x)=\frac{x}{x-2}$.
- $\lim_{x\to 0} x^2\cos\dfrac{1}{x}$
- Prove the addition law for limits.
- Find an equation of the line tangent to the curve $y=x^3-2x$ at $x=1$.
- $\lim_{t\to-\infty} (\sqrt{t^2-t+2}-\sqrt{t^2+t+1})$
- $\lim_{t\to\infty} \dfrac{e^x-1}{e^x+1}$
- $\lim_{x\to 0} \dfrac{x+1}{x^2(x-1)}$
- Find a formula for the inverse of $f(x)=x^2$, where the domain is all $x\le 0$.

Remember $\sin^2\theta+\cos^2\theta=1$, $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$, and $\sin x$ and $\cos x$ of $x=0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}$. The content from the precalculus review still applies.