Review for Math 1A Final

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Make sure to study the topics in "One-Minute Quickes for Midterm 1" and "Review for Midterm 2" as well.

1. Derive Newton's Method (by finding the x-intercept of the tangent line at a point $(x_i, f(x_i))$).

2. Define an antiderivative of a function. Prove that the difference between two antiderivatives on an interval is a constant (hence, if $F(x)$ is an antiderivative of f, then every antiderivative is of the form $F(x) + C$ with C a constant).

3. Remember an antiderivative for each of the following functions of x, where F, G are antiderivatives of f, g, respectively: x^n $(n \neq -1)$, $1/x$, $cf(x)$, $f(x) + g(x)$, e^x, $\cos x$, $\sin x$, $\sec^2 x$, $\sec x \tan x$, $1/\sqrt{1-x^2}$, $1/(1+x^2)$.

4. Write down the definition of the Riemann definite integral $\int_a^b f(t)\,dt$ of a continuous function f from a to b. Interpret the parts of the definition intuitively as areas of rectangles. Be aware of left- and right- Riemann integral formulations (though in the limit they give the same result).

5. Find the area between $y = \sin x$ and $y = 0$ on $[0, 2\pi]$. Make sure the areas are *positive*.

6. If you can stomache it, compute the definite integral $\int_0^3 (x^2 + x + 1)\,dx$ from the definition of the definite integral. You will need the sum formulas 5,6,7 on page 374.

7. Remember the following properties: $\int_a^b dx = b - a$, $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$, $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$. Interpret these geometrically.

8. Consider the following comparison test: if $f(x) \leq g(x)$ and $a \leq b$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$. Interpret this geometrically. What does this mean if f or g is constant 0? What does this mean if f or g is a constant? (These are example useful consequences.)

9. Fundamental Theorem of Calculus 1. If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of f. (In other words, antiderivatives always exist, and you can get at least one from the definite integral.)

10. Fundamental Theorem of Calculus 2. If f is continuous on $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$. (In other words, any antiderivative will do to compute a definite integral.) We sometimes write $F(b) - F(a)$ as $[F(x)]_a^b$ or just $F(x)]_a^b$.

11. Come up with and solve a problem involving the derivative of a function of x defined by a definite integral whose bounds are themselves functions of x.

12. Because of FTC1, we write $\int f(x)\,dx$ for the antiderivatives of f.

13. Net change theorem. $f(b) = f(a) + \int_a^b f'(x)\,dx$. Why is this true? Come up with physical examples of this theorem.

14. The substitution rule ("u-substitution"). If $u = g(x)$ is a differentiable function, then $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$. In practice, this means computing the implicit derivative $du = g'(x)\,dx$ and getting rid of x and dx by replacing dx with $du/g'(x)$ and replacing all resulting $g(x)$ with u. A trickier example is $\int_1^2 t^3\sqrt{t^2-1}\,dt$ (hint: t^3 is $t \cdot t^2$). A less tricky example is $\int \tan x\,dx$.

15. Integrals $\int_{-a}^{a} f(t)\,dt$ of odd or even continuous f.

16. If $f(x) \geq g(x)$ for all x in $[a, b]$, the area between the curves on that interval as an integral. Now the area between f and g if they may cross.

17. Interpretation of volume as a sum of thin elements (prisms, cylinders, "washers," "cylindrical shells").

18. How to compute the volume of revolution by both washers ($\pi(R(x)^2 - r(x)^2)\,dx$) and cylindrical shells ($2\pi r(x)h(x)\,dx$). (Note: there are some instances where, though one may be able to set up the volume as an integral by either method, only one has a computable antiderivative. Moral: be able to set up volumes both ways.) Be able to derive these volume elements from scratch, at least intuitively.