

Teaching statement. Kyle Miller

Simon Stevin, the 16th century polymath, was interested in making science, technology, and mathematics accessible to those outside the academy, and I have long appreciated how he translated the Latin *mathematica*, meaning knowledge or lessons, into his native Dutch: *wiskunde*, the art of what is certain. Beyond the remarkable assertion that there is anything to be certain about, it puts focus on the *how* in addition to the *what*. I believe that the goal of a mathematical education is to develop self-sufficiency in evaluating logical arguments and in clearly identifying what is and is not yet certain. Toward this end, students should learn mathematical knowledge, which gains them literacy in science, engineering, and mathematics, while simultaneously laying a solid foundation for a playground on which to practice the art.

For small classes, I like to create an environment that supports a constant conversation, since the feedback helps me calibrate ongoing discussion and it gives students the opportunity to practice speaking math. This requires establishing some amount of trust that my judgments of mathematical truths do not extend to judgments of the students themselves, and I succeed at least so far as having course reviews that consistently mention my approachability and willingness to dig into any detail, no matter how seemingly basic. One thing I have occasionally done, with careful planning, is to set ourselves a problem to work on together that is hard but which still allows for partial progress in a few directions — what I hope to show students is that frustration is normal and that, even if we fail to meet our goal, we can still extract new knowledge.

I try to organize recitation sections so that there is time for frequent informal one-on-one meetings. During group work, I rotate between groups asking questions about basic definitions, their answers, how they might validate their answers, how things would change if the problem was modified, and so on — I tend to use Polya-style heuristics in addition to these sorts of higher-level questions. Depending on the responses, I can turn attention to individual students to uncover specific gaps and misunderstandings or to sometimes give bits of enrichment to those who are interested. Meanwhile, this gives students practice to improve their fluency in spoken math. This process has made a number of quiet students comfortable to participate in class-wide discussions.

For larger classes, I run lectures in a traditional way, presenting an overview of the theory while complementing the textbook with additional intuitions, observations, and warnings, occasionally foreshadowing future material. Something students appreciate are the historical interludes I include to put math into context, and this gives opportunities to consider why math is the way it is. There is a temptation to cover too much in lecture, but when breadth and depth are in conflict, I lean toward depth, since the practice of deeper understanding is transferable.

For some concepts, either illustrations are not sufficient or experimentation is too laborious to be illuminating, so I create “math toys,” which are computer programs for students to play with. I have some for basis changes, eigenvalues, row reductions, complex roots of polynomials, mass-spring systems, and so on, and I provide guided tours for those who are uncomfortable with self-directed play. Students report they have gained intuition through these toys.

In a similar spirit, I am interested in developing a mathematics course based on the Lean proof assistant, which is used by many mathematicians for computer formalization of mathematics. Patrick Massot has successfully used it in a first-year undergraduate course on basic real analysis at Orsay, and Kevin Buzzard has supervised undergraduate mathematics research in Lean at Imperial College. In the process of proving things with Lean, you are presented with a view of what is currently known and unknown at all times, and it gives hands-on experience with mathematical truth. This frees students from being uncertain about what is an acceptable proof, and the interactive feedback has been reported to give undergrads a deeper understanding of math.